Creation of an infinite Fibonacci Number Sequence Table

(Weblink to the **Infinite Fibonacci Number Sequence Table**)

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Abstract :

A Fibonacci-Number-Sequences-Table was developed, which contains infinite Fibonacci-Sequences. This was achieved with the help of research results from an extensive botanical study. This study examined the phyllotactic patterns (Fibonacci-Sequences) which appear in the tree-species "Pinus mugo" at different altitudes (from 550m up to 2500m) With the increase of altitude above around 2000m the phyllotactic patterns change considerably, the number of patterns (different Fibonacci Sequences) grows from 3 to 12, and the relative frequency of the main Fibonacci Sequence decreases from 88 % to 38 %. The appearance of more Fibonacci-Sequences in the plant clearly is linked to environmental (physical) factors changing with altitude. Especially changes in temperature- / radiation- conditions seem to be the main cause which defines which Fibonnacci-Patterns appear in which frequency.
The developed (natural) Fibonacci-Sequence-Table shows interesting spatial dependencies between numbers of different Fibonacci-Sequences, which are connected to each other, by the golden ratio (constant Phi) → see Table
An interesting property of the numbers in the main Fibonacci-Sequence F1 seems to be, that these numbers contain all prime numbers as prime factors ! in all other Fibonacci-Sequences ≥ F2, which are not a multiple of Sequence F1, certain prime factors seem to be missing in the factorized Fibbonacci-Numbers (e.g. in Sequences F2, F6 & F8).
With the help of another study (Title: Phase spaces in Special Relativity: Towards eliminating gravitational singularities) a way was found to express (calculate) all natural numbers and their square roots only by using constant Phi (φ) and 1. An algebraic term found by Mr Peter Danenhower, in his study, made this possible. With the formulas which I found, it seems to be possible to eliminate number systems and base mathematics only on Phi (φ) and 1 (see my 12 conjectures)

Introduction :

In botany Phyllotaxis describes the arrangement of leaves on spiral paths on the stem of a plant. Phyllotactic spirals form a distinctive class of patterns in nature. But the true cause of these phyllotactic spirals, which appear everywhere in nature, still isn't found yet ! The current believe ist that the spiral patterns of leaves on the stem of a plant, which can be explained and described by Fibonacci Number Sequences, is controlled by plant hormones like Auxin.

However this can't be the true cause for the precise Fibonacci-spiral-patterns seen on plants ! Because the extensive botanical study carried out by Dr. Iliya Vakarelov clearly shows that the Fibonacci-spiral formation is influenced by environmental conditions, especially temperature and radiation (light).
Therefore the Fibonacci-spiral formation seems to have a fundamental physical cause ! Dr. Vakarelov's study also showed that the phyllotactic-patterns changed cyclic, with six year duration of the cycles. I

I have written an own **hypothesis about the cause of phyllotactic (Fibonacci) patterns** :
see study : → **Microscope Images indicate that Water Clusters are the cause of Phyllotaxis** , alternative: **Weblink 2**
Please also have a look at this study : → **EHT2017 may provide evidence for a Poincare Dodecahedral Space Universe**

Contents Page

1. **Extracts from a study produced by Dr. Iliya Iv. Vakarelov**, University of Forestry, Bulgaria (1982-1994)

<u>Title :</u> "Changes in phyllotactic pattern structure (Fibonacci Sequences) in Pinus mugo due to changes in altitude "

from the book „Symmetry in Plants" by Roger V. Jean and Denis Barabe, Universities of Quebec and Montreal, Canada (Part I. – Chapter 9 , pages 213 – 229) , **weblinks :** **Weblink 1** (Google Books), **Weblink 2**

<u>Fig :</u> *Pinus mugo*

Research Site and methods :

Pinus Mugo grows in high mountainous parts at altitudes up to 2500m forming vast communities. The vertical profile of the research sites for *Pinus mugo* was situated along the northern slopes of the eastern part of the Rila mountain, and test specimens were collected from the following altitudes : 1900, 2200 and 2500 m. Test specimens were also collected from the city of Sofia (at 550 m) where *Pinus mugo* is grown as decorative plant.
The research was carried out over a period of 12 years (except of altitude 550m here research was carried out only around 6 years). The initation of leaf primordia in the bud (meristem) occurs at the end of the growing period. The apical meristem of *Pinus mugo* starts this process around the beginning of mid of August and ends in autumn when the air temperature goes below a certain point.

<u>The interesting results of the study :</u>

(3) With the increase of altitude from 1900m to 2500m the phyllotactic pattern structure of "Pinus mugo" twigs changes considerably, the number of patterns (different Fibonacci Sequences) grows from 3 to 12, and the relative frequency of the main sequence decreases from 88 % to 38 %.

At the upper boundary of Pinus mugo natural distribution – at about 2500m, the variation of phyllotactic twig pattern structure (entropy) becomes cyclic, with six year duration of the cycles.

(5) The changes in temperature during the period of phyllotactic pattern formation of Pinus mugo twigs determine about 48 % of the changes in pattern structure, the latter lagging behind with one or two years.

It is obvious that when the altitude increases, the number of phyllotactic patterns (Fibonacci-Sequences) of the vegetative organs of *Pinus mugo* also increases above a given altitude. → see Table below !

Sequence No.	FIBONACCI–Sequences present in given altitude	Altitute in (m)									
		550		1900		2200		2500		Total	
		Frequency	Relative Frequency	Frequency	Relative Frequency	Frequency	Relative Frequency	Frequency	Relative Frequency	Frequency	Relative Frequency
F1	⟨1,2,3,5,8,13,...⟩	231	0.902	431	0.885	619	0.812	246	0.381	1527	0.710
F3	2⟨1,2,3,5,8,13,...⟩	16	0.063	34	0.070	35	0.046	111	0.172	196	0.092
F2	⟨1,3,4,7,11,18,...⟩	3	0.012	22	0.045	49	0.064	86	0.133	160	0.074
F4	3⟨1,2,3,5,13,...⟩	6	0.023	–	–	29	0.038	98	0.152	133	0.062
F8	⟨2,5,7,12,19,31,...⟩	–	–	–	–	10	0.013	50	0.077	60	0.028
F11	⟨3,7,10,17,27,44,...⟩	–	–	–	–	5	0.007	18	0.028	23	0.011
F6	⟨1,4,5,9,14,23,...⟩	–	–	–	–	1	0.001	8	0.012	9	0.004
F9	2⟨1,3,4,7,11,18,...⟩	–	–	–	–	4	0.005	7	0.011	11	0.005
(?) F6	⟨1,7,8,15,23,38,...⟩	–			–	2	0.003	7	0.011	9	0.004
F5	4⟨1,2,3,5,8,13,...⟩	–			–	8	0.011	9	0.013	17	0.008
(?) F13	⟨1,6,7,13,20,33,...⟩	–			–	–	–	3	0.005	3	0.001
F10	⟨2,7,9,16,25,41,...⟩	–	–	–	–	–	–	3	0.005	3	0.001

<u>Note</u> : The number of Fibonacci-Sequences is **increasing** with altitude !

<u>Table 1</u> : Data on the frequency and relative frequency of the different phyllotactic patterns for ***Pinus mugo*** twigs at different altitudes. Specimen formed during the period 1982-1994 have been tested for all sites except for the one at 550 m where the period covers the years 1989 – 1993.

1.1 Different Temperatures at different altitudes caused changes in Phyllotactic-pattern-variation

Different temperatures at the research sites at different altitudes (550 – 2500 m), during the period of **phyllotactic-pattern** formation, caused the changes in variability of the found phyllotactic patterns.
The number of found patterns (different Fibonacci Sequences) increased with altitude. But because „temperature at different altitudes" is a complex subject, **we must understand „temperature & radiation at different altitudes" precisely**, to understand the causes of pattern variability ! **→ see also my study : Weblink 1**

Some fundamental facts about „Temperature" :

The temperature (thermal energy) of a solid body (e.g. a plant) is associated primarily with the vibrations of it's molecules. Heat transfer to the plant happens through thermal conduction or thermal radiation. **Here especially heat transfer through thermal radiation to the plant must be examined more closely !** This is the transfer of energy by means of eloctromagnetic waves (photons). Especially **Infrared-Radiation** is important for the heat transfer to the plant

Infrared radiation lies energetically in the area of the rotation niveaus of small molecules and in the area of the oscillation niveaus of molecule bindings. That means the absorption of infrared light (infrared radiation) leads to an vibration excitation of the molecule bindings and of the matter in the plant in general, or in other words to an increase of the heat energy (temperature) of the plant. The energetic **Near-Infrared-Radiation (IR-A/B)**, with approximately **0.7 to 3 µm** wavelength can excite **overtone or harmonic vibrations** in matter (in the plant molecules / plant structure)

1.2 Radiation is different at different altitudes

The temperature (thermal energy) of the plant increases or decreases by absorbing (see Spectroscopy) or by emitting radiation, or through thermal conduction.
Especially **Near-Infrared-Radiation** with wave-lengths of **0.7 to 3 µm** is absorbed by the water molecules of the plant and is responsible for the temperature of the plant
The **distribution of Infrared-Radiation in the atmosphere is different in different altitudes**, as the diagram on the right clearly shows. The sun's **IR-A/B-radiation with 1 to 3 µm wave-lengh is absorbed** by H_2O, CO_2 and other atmospheric gas, more and more **on it's way from 10 km altitude to sealevel.** But also **IR-C** and **Far-IR radiation** with **3-50 µm** gets absorbed more & more.

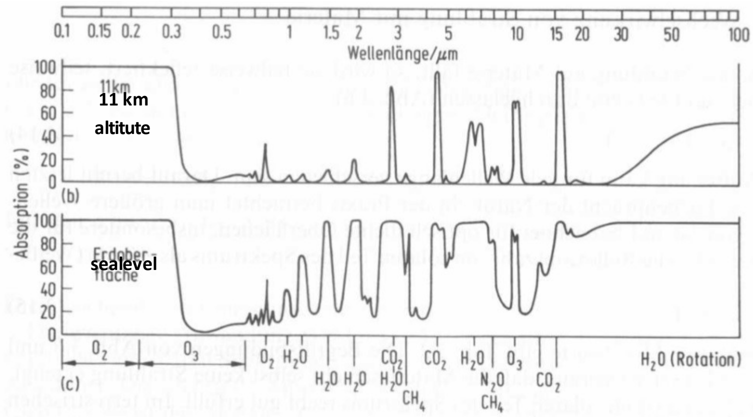


Fig. 2 : **Distribution of radiation in the atmosphere, at 11 km altitude and at sealevel. It is obvious that at higher altitude the variation of radiation with different wave lengths is higher than at sea level**

Another important result ot Dr. Vakarelov's study :

Additional Dr. Vakarelov's study showed that the **phyllotactic pattern variability (Fibonacci Sequence-variabiliy) changed over the years !**
The study also showed that **the variability of the phyllotactic patterns in high altitude changed cyclic, with six year duration of the cycles.**

Fig. 3

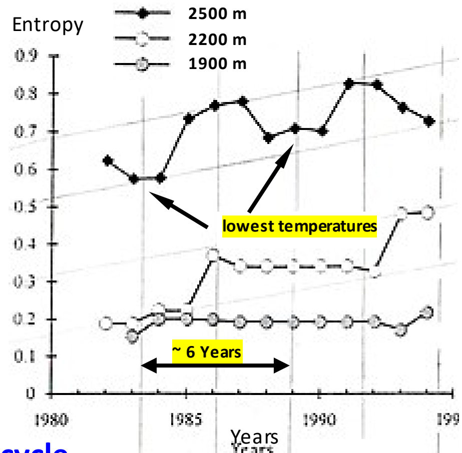


Figure 3 : The diagram on the righthand side shows the variability of entropy (variability of Fibonacci Sequences) with respect to altitude for „Pinus Mugo" twigs. It is obvious that **at 2500 m** the curve shows a clear **cyclic process,** while at 2200 m the cyclic process is less significant, and at 1900 m nonexistent. **The cyclic process has a period of ~6 Years.**

1.3 Phyllotactic-pattern-variability seems to vary with the sunspot-cycle

Figure 4 : The next diagram on the right shows how **sunspot-numbers**, cosmic ray flux, X-ray's and proton flux changes with the 11 to 12 year sunspot-cycle. A weak correlation between **phyllotactic-pattern-variability** and cosmic ray flux is noticable.

How does the radiation in the atmosphere change with the sunspot-cycle ? :
Solar **X-ray radiation** and **Ultraviolet radiation** (especially **extreme UV (EUV)** with **10 to 124 nm** wavelength varies markedly over the sunspot-cycle (**UV-B** at **300 nm** (by up to 400% !). This radiation has a big impact on Earth's upper atmosphere. Increased X-ray & UV-radiation leads to heating of the Ionosphere. The ionisation of the Ionosphere also affects the propagation of radio-waves. Especially the HF-radio spectrum (3-30 MHz), but also the MF- & VHF-radio-spectrum is effected (MF=300kHz-3MHz & VHF=30-100 MHz). 30 MHz corresponds to 10 m wave-length.

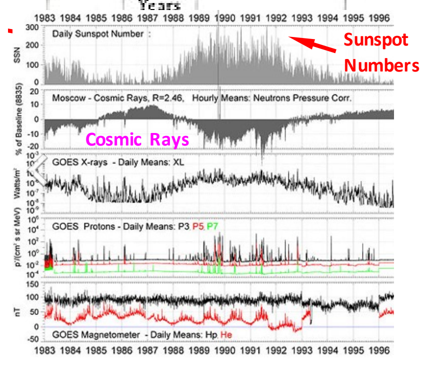


Fig. 4: see: Sun-Climate-Connections

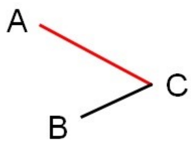
2 From the Fibonacci-Sequences shown by *Pinus mugo* at 2500m an infinite Fibonacci-Table was developed :

There are clear spatial interdependencies noticable between the different Fibonacci-Sequences, which are connected by the golden ratio **φ** . There is a complex network visible between the numbers of all Sequences. This table of Fibonacci-Number Sequences can be extended towards infinity and all natural numbers are contained in the lower half only once!

For 3 numbers A, B and C in the below shown arrangement, which belong to the same 3 (or 2) different Fibonacci-Sequences, the following rule is true :

The ratio of the difference (C-A) indicated by a "red line", to the difference (B-C) indiated by a "black line" is approaching the golden ratio **φ** for the further progressing Fibonacci-Number Sequences towards infinity (downwards in the table).

„**Main Bow-Structures**" are also linked by the „golden ratio" **φ** !



$$\rightarrow \quad \lim \frac{C-A}{B-C} = \varphi \quad \text{for } A, B, C \rightarrow \infty$$

FIBONACCI – Number Sequences No. 1 to 14 (F1 - F14) → see extended table in the Appendix !

Row No.	F1 Fibonacci-Base-Sequence	F2 Lucas-Sequence	F3 Fibonacci-Sequence (x 2)	F4 Fibonacci-Sequence (x 3)	F5 Fibonacci-Sequence (x 4)	F6	F7	F8	F9 Lucas-Sequence (x 2)	F10	F11	F12	F13 Lucas-Sequence (x 3)	F14
1	1	1				1	1							
2	2		2					2	2	2				
3	3	3		3							3	3	3	
4		4	4		4	4								4
5	5					5	5	5						
6			6	6			6		6					
7		7						7		7	7			
8	8				8				8			8		
9				9		9				9			9	9
10			10								10			
11		11					11					11		
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58									58					
59														
60						60								

3 5 7 8 11 13

Lucas-Sequence (bottom main bows)

Note : Below this line all natural numbers are contained in the Fibonacci Sequences just once !

Fibonacci-Sequence (top main bows)

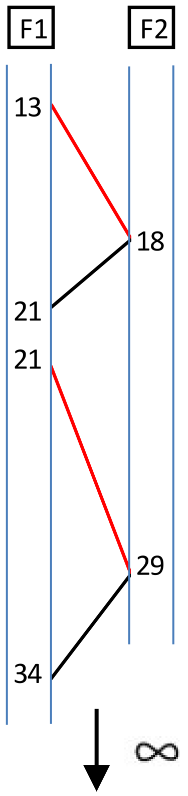
∞

3 A general rule exists which connects numbers of different Fibonacci-Sequences by the golden ratio φ

➔ **The following two examples explain the rule which was described in general on the previous page :**

The examples show how the quotient of the differences between the numbers of designated Fibonacci-Sequences (indicated by red- and black-lines in the table), is approaching the golden ratio for the number sequences progressing towards infinity.

For the examples we look at the Fibonacci Sequences **F1, F2** and **F3** (→ F2 is the Lucas-Sequence, F3 = F1 x 2)

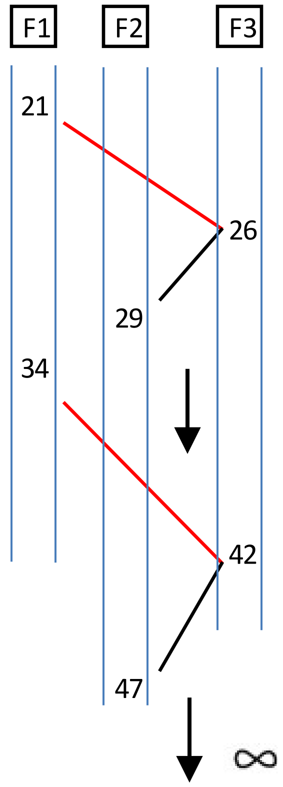


Note : The differences correspond to numbers of the **F1–main sequence**

$$\frac{18 - 13}{21 - 18} = \frac{5}{3} = 1{,}666$$

$$\frac{29 - 21}{34 - 29} = \frac{8}{5} = 1{,}6$$

φ



Note : The differences correspond to numbers of the **F1–main sequence**

$$\frac{26 - 21}{29 - 26} = \frac{5}{3} = 1{,}666$$

$$\frac{42 - 34}{47 - 42} = \frac{8}{5} = 1{,}6$$

φ

4 Interesting properties of the Fibonacci-F1 Sequence (and other Fibonacci-Sequences) :

- The numbers of the **Fibonacci F1** – Number Sequence seem to contain all prime numbers as prime factors !
- This is not the case for all other Fibonacci-Sequences where certain prime factors are missing ! (**see Appendix**)
- And all prime factors appear periodic in defined "number-distances" in the sequence (see left side of table)
- This is the case for all Fibonacci-Sequences ! (→ These mentioned properties must be analysed in more detail !)

Table 2 : Periodicity of the prime factors of the **Fibonacci F1** - Number Sequence :

some prime factors shown in table form													in prime factors factorized Fibonacci-Numbers		sum of digits	Fibonacci-Sequence F1			
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products	new products		F	F'	F''	Nr.
															1	1			1
															1	1			2
															2	2	1		3
															3	3	1		4
															5	5	2	1	5
												2^3		2x2x2	8	8	3	1	6
															4	13	5	2	7
									7		3			3x7	3	21	8	3	8
						17						2		2x17	7	34	13	5	9
								11		5				5x11	10	55	21	8	10
															17	89	34	13	11
											3^2	2^4	2x2x2x	3x3	9	144	55	21	12
															8	233	89	34	13
			29				13							13x29	17	377	144	55	14
										5		2		2x5x61	7	610	233	89	15
									7		3		3x7x	47	24	987	377	144	16
															22	1597	610	233	17
					19	17						2^3	2x17x	2x2x19	19	2584	987	377	18
	37													37x113	14	4181	1597	610	19
41								11		5	3		5x11x	3x41	24	6765	2584	987	20

➔ **See some selected Fibonacci-Sequences in more detail in the Appendix !**

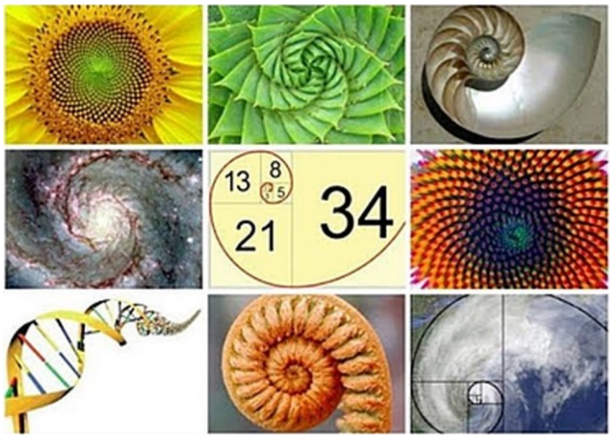
5 Constant φ (Φ) defines all Fibonacci-Sequences and the square roots of all natural numbers

The asymptotic ratio of successive Fibonacci numbers leads to the Golden Ratio constant φ (or Φ)
The Fibonacci Sequences describe morphological patterns in a wide range of living organisms. It is one of the most remarkable organizing principles mathematically describing natural and man made phenomena.

The constant φ is the positive solution of the following quadratic equation :

$$x + 1 = x^2$$

$$\rightarrow \quad \varphi = \frac{1+\sqrt{(5)}}{2} = 1.618034...$$

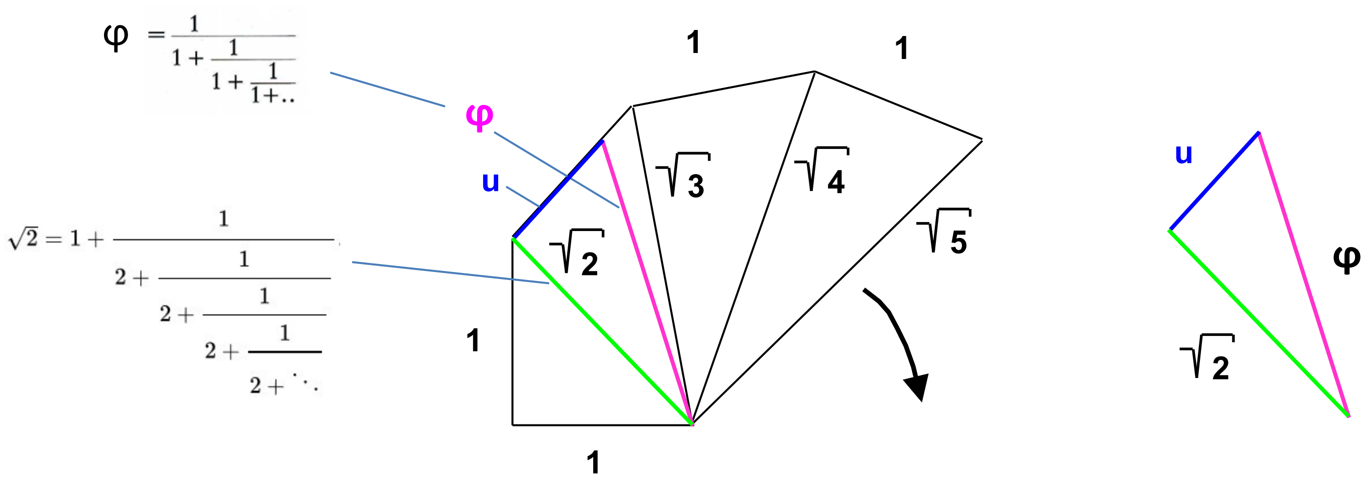


Because the value of constant φ is close to the **square root of 2** and the **square root of 3**, I draw φ into the start section of the **Square Root Spiral :**

5.1 To the discovery of an important algebraic equation regarding Constant φ (Phi)

→ This discovery indicates that constant φ and the base unit **1** form the base of mathematics and geometry.
And the distribution and structure of matter (energy) in space, is fundamentally based on constant Phi and 1

The start of the Square Root Spiral is shown with the constant φ drawn in :



Now we see what we can do with this arrangement of right triangles , and with the help of the Pythagorean theorem.

From the right triangle φ , square root of 2 & u follows :

$\varphi^2 = (\sqrt{2})^2 + u^2$; application of the Pythagorean theorem

$\rightarrow \quad u = \sqrt{\varphi^2 - 2} = 0{,}786151377.....$; we can calculate this value of **u** with the calculator

I did research with Google, and I found a study where the constant u was expressed with an algebraic term !

With the help of this algebraic term it was possible to find interesting new properties of constant φ !

→ **see next page !**

The algebraic calculation of the square roots of all natural numbers only with constant φ & 1

From Equation (4.10) from the study shown on the righthand side I have found the algebraic term which describes the calculated value of u :

$$\frac{\sqrt{2\sqrt{5}-2}}{2} = 0{,}786151377\ldots = u$$

From this algebraic term it follows :

PHASE SPACES IN SPECIAL RELATIVITY : TOWARDS ELIMINATING GRAVITATIONAL SINGULARITIES

from PETER DANENHOWER → see weblink : https://arxiv.org/pdf/0706.2043.pdf

Abstract : This paper shows one way to construct phase spaces in special relativity by expanding Minkowski Space. These spaces appear to indicate that we can dispense with gravitational singularities. The key mathematical ideas in the present approach are to include a complex phase factor, such as, eιφ in the Lorentz transformation and to use both the proper time and the proper mass as parameters. To develop the most general case, a complex parameter σ = s + im, is introduced, where s is the proper time, and m is the proper mass, and σ and σ /|σ| are used to parameterize the position of a particle (or reference frame) in space-time-matter phase space. A new reference variable, u = m/r , is needed (in addition to velocity), and assumed to be bounded by 0 and c^2/G = 1, in geometrized units. Several results are derived: The equation E = mc^2 apparently needs to be modified to $E^2 = (s^2c^{10})/G^2 + m^2c^4$, but a simpler (invariant) parameter is the "energy to length" ratio, which is c^4 /G for any spherical region of space-time-matter. The generalized "momentum vector" becomes completely "masslike" for **u ≈ 0.7861…, which we think indicates the existence of a maximal gravity field. Thus, gravitational singularities do not occur**.
Instead, as u → 1 matter is apparently simply crushed into free space. In the last section of this paper we attempt some further generalizations of the phase space ideas developed in this paper.

$$\boxed{\sqrt{\varphi^2 - 2} = \frac{\sqrt{2\sqrt{5}-2}}{2}}$$; **now we can equate the two algebraic terms which represent the same constant !**

$$\rightarrow \quad 4\varphi^2 - 8 = 2\sqrt{5} - 2$$; we square both sides and transform

$$\varphi^2 = \frac{\sqrt{5}+3}{2}$$; (1) we solve for φ^2

Compare ! ⟷ $\varphi = \frac{\sqrt{5}+1}{2}$

$$\sqrt{5} = 2\varphi^2 - 3$$; (2) we solve for $\sqrt{5}$

Now we go back to the square root spiral and use the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2$$; application of the Pythagorean theorem

$$6 = (2\varphi^2 - 3)^2 + 1$$; we replace $\sqrt{5}$ by equation (2) and transform

$$\rightarrow \quad 3 = \frac{\varphi^4 + 1}{\varphi^2} \quad (3) \quad \rightarrow \quad \sqrt{3} = \sqrt{\frac{\varphi^4 + 1}{\varphi^2}} \quad (4)$$; square root 3 expressed by φ and 1 !

Now we use the following right triangle :

$$(\sqrt{3})^2 = (\sqrt{2})^2 + 1^2$$; application of the Pythagorean theorem & inserting equation (3)

$$\rightarrow \quad 2 = \frac{\varphi^4 + 1}{\varphi^2} - 1 \quad \rightarrow \quad 2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} \quad (5) \quad \text{and} \quad \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \quad (6)$$

square root 2 expressed by φ and 1 !

Now we insert equation (3) in equation (2) :

$$\rightarrow \quad \sqrt{5} = 2\varphi^2 - \frac{\varphi^4 + 1}{\varphi^2} \quad \rightarrow \quad \sqrt{5} = \frac{\varphi^4 - 1}{\varphi^2}$$; (7) ; square root 5 expressed by φ and 1 !

Now we use the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2$$; application of the Pythagorean theorem & inserting equation (7)

$$\rightarrow \quad 6 = \left(\frac{\varphi^4 - 1}{\varphi^2}\right)^2 + 1 \quad \rightarrow \quad 6 = \frac{\varphi^8 - \varphi^4 + 1}{\varphi^4} \quad (8) \quad \text{and} \quad \sqrt{6} = \sqrt{\frac{\varphi^8 - \varphi^4 + 1}{\varphi^4}} \quad (9)$$

We can now continue and use the following right triangles of the square root spiral :

$$(\sqrt{7})^2 = (\sqrt{6})^2 + 1^2$$; application of the Pythagorean theorem & inserting equation (8)

$$\rightarrow \quad 7 = \frac{\varphi^8 + 1}{\varphi^4} \quad (10) \quad \rightarrow \quad \sqrt{7} = \sqrt{\frac{\varphi^8 + 1}{\varphi^4}} \quad (11)$$

In the same way we can now calculate all square roots of all natural numbers with the next right triangles :

$$\rightarrow \quad 8 = \frac{\varphi^8 + \varphi^4 + 1}{\varphi^4} \quad (12) \quad \text{and} \quad \sqrt{8} = \sqrt{\frac{\varphi^8 + \varphi^4 + 1}{\varphi^4}} \quad (13)$$

$$\rightarrow \quad 10 = \frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4} \quad (14) \quad \text{and} \quad \sqrt{10} = \sqrt{\frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4}} \quad (15)$$

$$\rightarrow \quad 11 = \frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4} \quad (16) \quad \text{and} \quad \sqrt{11} = \sqrt{\frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4}} \quad (17)$$

$$\rightarrow \quad 12 = \frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4} \quad (18) \quad \text{and} \quad \sqrt{12} = \sqrt{\frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4}} \quad (19)$$

From the above shown formulas (equations) I have realized a general rule for all natural numbers > 10 :

Note : → The expression (3+n) in the rule can be replaced by products and / or sums of the equations (3) to (13)

$$\rightarrow \quad (10 + n) = \frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4} \quad (20) \quad \text{and} \quad \sqrt{(10+n)} = \sqrt{\frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4}} \quad (30)$$

For $n \rightarrow \infty$

With this general formula we can express all natural numbers ≥ 10 and their square roots only with φ and 1 !

This statement is also valid for all rationals (fractions) and their square roots. This is a quite interesting discovery !!

Constant Phi (φ) which defines the structure of the Dodecahedron and Icosahedron (together with base unit 1) is a very important (space structure) constant for the real / physical world ! Please also read my following study :

→ **The Black Hole in M87 (EHT2017) may provide evidence for a Poincare Dodecahedral Space Universe**

Weblink 1 to the study : http://vixra.org/abs/1907.0348 ; alternative : **Weblink 2 :** Weblink to_archive.org

Constant Pi (π) can also be expressed by only using constant φ and 1 !

Viete's formula from 1593 :

$$\pi = \frac{2}{\sqrt{2}} \frac{2}{\sqrt{2+\sqrt{2}}} \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \cdots$$

→ It is also possible to derive from Viète's formula a related formula for π that still involves nested square roots of two, but uses only one multiplication :

$$\pi = \lim_{k\to\infty} 2^k \underbrace{\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2}}}}}}}_{k \text{ square roots}}$$

If we replace the number 2 in the above shown formulas by the found equation (5) where number 2 can be expressed by constant φ and 1, then we can express the constant Pi (π) also by only using the constant φ and 1 !

Replace Number 2 in the above shown formulas with this term.

$$\rightarrow \quad 2 = \frac{\varphi^4 + 1}{\varphi^2} - 1 \quad \rightarrow \quad \boxed{2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \quad (5) \quad \text{and} \quad \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \quad (6)$$

It becomes clear that the irrationality of Pi (π) is also only based on the constant φ and 1, in the same way as the irrationality of all irrational square roots, is only based on constant φ & 1 ! Numbers don't exist ! Only φ & 1 exist !

Constant Pi (π) can now be expressed in this way, by only using constant φ and 1 :

$$\pi = \lim_{k\to\infty} \left[\frac{\varphi^4-\varphi^2+1}{\varphi^2}\right]^k \underbrace{\sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2} - \sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2} + \sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2} + \cdots + \sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2}}}}}}_{k \text{ square roots}}$$

It becomes clear that the irrationality of Pi (π) is only based on constant φ and 1, in the same way as the irrationality of all irrational square roots, is only based on constant φ & 1 !

Natural Numbers, their square roots and irrational and transcendental constants like Pi (π) can be expressed (calculated) by only using constant φ and 1 ! This is also valid for all rationals (fractions) and their square roots.

Numbers and number-systems don't seem to exist ! They are manmade and therefore can be eliminated.

This is an interesting discovery because it allows to define most (maybe all) geometrical objects only with φ & 1 !

The result of this discovery may lead to a new base of number theory. Not numbers like 1, 2, 3,….. and constants like Pi (π) etc. are the base of Number Theory ! Only the constant φ and the base unit 1 (which shouldn't be considered as a number) form the base of mathematics and geometry. This will certainly also have an impact on Physics !

Constant φ and the base unit 1 must be considered as the fundamental „space structure constants" of the real physical world !

In the physical world the geometries of all possible crystal-lattice-structures are fundamentally based on Phi and 1.

There probably isn't something like a base unit if we consider a „wave model" as the base of physics and if we see the universe as one oscillating unit. In the universe everyting is connected with everything. see : Quantum Entanglement

→ **Please also read my 12 Conjectures on the next page (Chapter 6)**

Chapter 6 :

Referring to my discovery regarding constant φ (Phi), I want to define the following 12 Conjectures :

<u>Here the 12 conjectures</u> : (→ you can call them **Harry K. Hahn's conjectures**)

1.) All Natural Numbers and their square roots can be expressed (calculated) by only using the mathematical constant Phi (golden mean = 1.618..) and number 1. This statement is also valid for all rationals (fractions) and their square roots

2.) All existing irrational numbers seem to be constructions out of Phi and 1.
For example the irrational transcendental constant Pi (3.1415926....) can also be expressed by only using Phi and 1 !

3.) Phi and 1 are the base units of Mathematics ! Numbers and number-systems don't exist ! They are manmade and therefore can be eliminated. In principle Mathematical Science can be carried out by only using Phi and 1, as base units.

4.) All geometrical objects, including the Platonic Solids can be described by only using constant Phi and 1.
Because all natural numbers, their square roots, rationals (fractions) and probably all irrational and all transcendental numbers too, can be expressed by only using Phi and 1.

5.) Point 4.) leads me to the conclusion that in the physical world the geometries of all possible crystal-lattice-structures are fundamentally based on Phi and 1. The more fundamental the lattice the simpler it can be expressed by Phi and 1.

6.) Point 4.) 5.) & 7.) leads me to the conclusion that on the molecular level (and probably on the atomic level too), as well as on the macroscopic (cosmic) level the distribution and structure of matter (=energy) in space, is fundamentally based on constant Phi and 1. → **Phi represents a fundamental physical „Space Structure Constant"**
Together with Point 7.) this indicates that the curvature of spacetime at the molecular level (crystals) and at the atomic level, as well as on the macroscopic level is defined only by the "Space Structure Constant Phi" and the base unit 1.
→ This idea will help to unify General Relativity with Quantum Mechanics ! If the gravitational singularity in M87 indeed has a dodecahedral structure then gravitation, which is the geometric property of spacetime, can be described in Quantum Mechanics and at the cosmic level by the same constant duo : Phi and base unit 1 !

7.) The structure of the M87 black hole (→ **EHT2017**) indicates a dodecahedral structure. The distribution of matter in gravitational singularities therefore seems to be defined essentially by constant Phi and base unit 1 ! The largescale distribution of matter in the universe seems to be predominantly based on an order-5 Poincare-Dodecahedral-Space.
→ <u>weblink to my study</u> (or alternatively here : <u>http://vixra.org/abs/1907.0348</u>)
Title : "EHT2017 may provide evidence for a Poincare Dodecahedral Space Universe"

8.) The natural numbers can be assigned to a defined infinite set of Fibonacci-Number Sequences.

9.) This infinite set of Fibonacci-Number Sequences , and the numbers contained in these sequences, are connected to each other by a complex precisely defined spatial network based on constant Phi. (**→ see table in Appendix A**). For the progressing Fibonacci-Sequences towards infinity, the connections between the numbers approach constant Phi.
→ see explanation in Chapter 2 and 3 and in Appendix A

10.) Constant Phi (golden mean = 1.618..) must be a fundamental constant of the final equation(s) of the universal mathematical and physical theory. (→ It may be the only irrational constant that appears in the(se) equation(s))

11.) The number-5-oscillation (→ the numbers divisible by 5) in the two number sequences 6n+5 (Sequence 1) and 6n+1 (Sequence 2), with n=(0,1,2,3,...) , defines the distribution of the prime numbers and non-prime-numbers. The number-5-oscillation defines the starting point and the wave length of defined non-prime-number-oscillations in these Sequences 1+2 (SQ1 & SQ2). (Note : the combination of the two sequences SQ1 & SQ2 is considered here)
→ weblink to my study : <u>https://arxiv.org/abs/0801.4049</u> (or alternatively here : <u>http://vixra.org/abs/1907.0355</u>)
For a quick overview please see **pages 15 to 18** in this study : <u>weblink to the study</u> : **"EHT2017 may provide evidence..."**

12.) The importance of the number-5-oscillation for the distribution of primes and non-primes is a further indication for the conjecture that the largescale structure of the universe seems to be predominantly (mainly) based on an order-5 Poincare-Dodecahedral-Space structure. → The space structure of the universe seems to be based essentially on the **5.Platonic Solid : the Dodecahedron** (→ consisting of 12 regular pentagonal faces, three faces meeting at each vertex)

The time will show if my Conjectures are correct !

References :

Symmetry in Plants - by Roger V. Jean & Denis Barabe (1998) – University Quebec, CA - **ISBN No. : 981-02-2621-7**
Weblink (Google Books) : https://books.google.de/books/about/Symmetry_In_Plants.html?id=2fbsCgAAQBAJ&redir_esc=y

Changes in phyllotactic pattern structure in Pinus mugo due to changes in altitude
Study to Fibonacci pattern variation in Pinus Mugo by Dr. Iliya Iv. Vakarelov, University of Forestry, Bulgaria (1982-1994)
From the book „**Symmetry in Plants**" by Roger V. Jean and Denis Barabe, Universities of Quebec and Montreal, Canada (Part I. Chapter 9 , pages 213-229) , ISBN : 981-02-2621-7 , **Weblinks : Weblink_1 ; Weblink_2** (Google Books)

Other studies which indicate phyllotactic pattern variability (with a noticeable distribution pattern) within the same species → in all probability depending mainly on environmental factors :

Aberrant phyllotactic patterns in cones of some conifers : a quantitative study - by Veronika Fierz
Weblink : Aberrant phyllotactic patterns in cones of some conifers (researchgate.net)

Novel Fibonacci and non-Fibonacci structure in the Sunflower - by J. Swinton, E. Ochu & Others
https://www.researchgate.net/publication/303354855_Novel_Fibonacci_and_non-Fibonacci_structure_in_the_sunflower ; **Weblink2**

A study which indicates that far-red & infrared radiation with wave-lengths > 750 nm is the trigger for phyllotactic-pattern formation & bud-induction :

Red Light Affects Flowering under long days in a Short-day Strawberry Cultivar
by Fumiomi Takeda & D. Michael Glenn - USDA-ARS, Appalachian Fruit Research Station (USA), Kearneysville, WV 2543 0 – publication: HortScience 43(7):2245-2247.2008 - **Weblinks to study :** Weblink 1 , Weblink 2

To the importance of constant Phi (φ) for the physical world , and studies regarding the Square Root Spiral :

Phase Spaces in Special Relativity : Towards eliminating Gravitational Singularities
by Peter Danenhower , Weblink : https://arxiv.org/pdf/0706.2043.pdf

Microscope Images indicate that Water Clusters are the cause of Phyllotaxis - by Harry K. Hahn
https://archive.org/details/microscope-images-indicate-that-water-clusters-are-the-cause-of-phyllotaxis
alternative weblink : https://vixra.org/abs/2005.0118

The Black Hole in M87 (EHT2017) may provide evidence for a Poincare Dodecahedral Space Universe - by Harry K. Hahn
https://archive.org/details/TheBlackHoleInM87EHT2017MayProvideEvidenceForAPoincareDodecahedralSpaceUniverse/page/n1
alternative Weblink : http://vixra.org/abs/1907.0348

The golden ratio Phi (φ) in Platonic Solids : http://www.sacred-geometry.es/?q=en/content/phi-sacred-solids

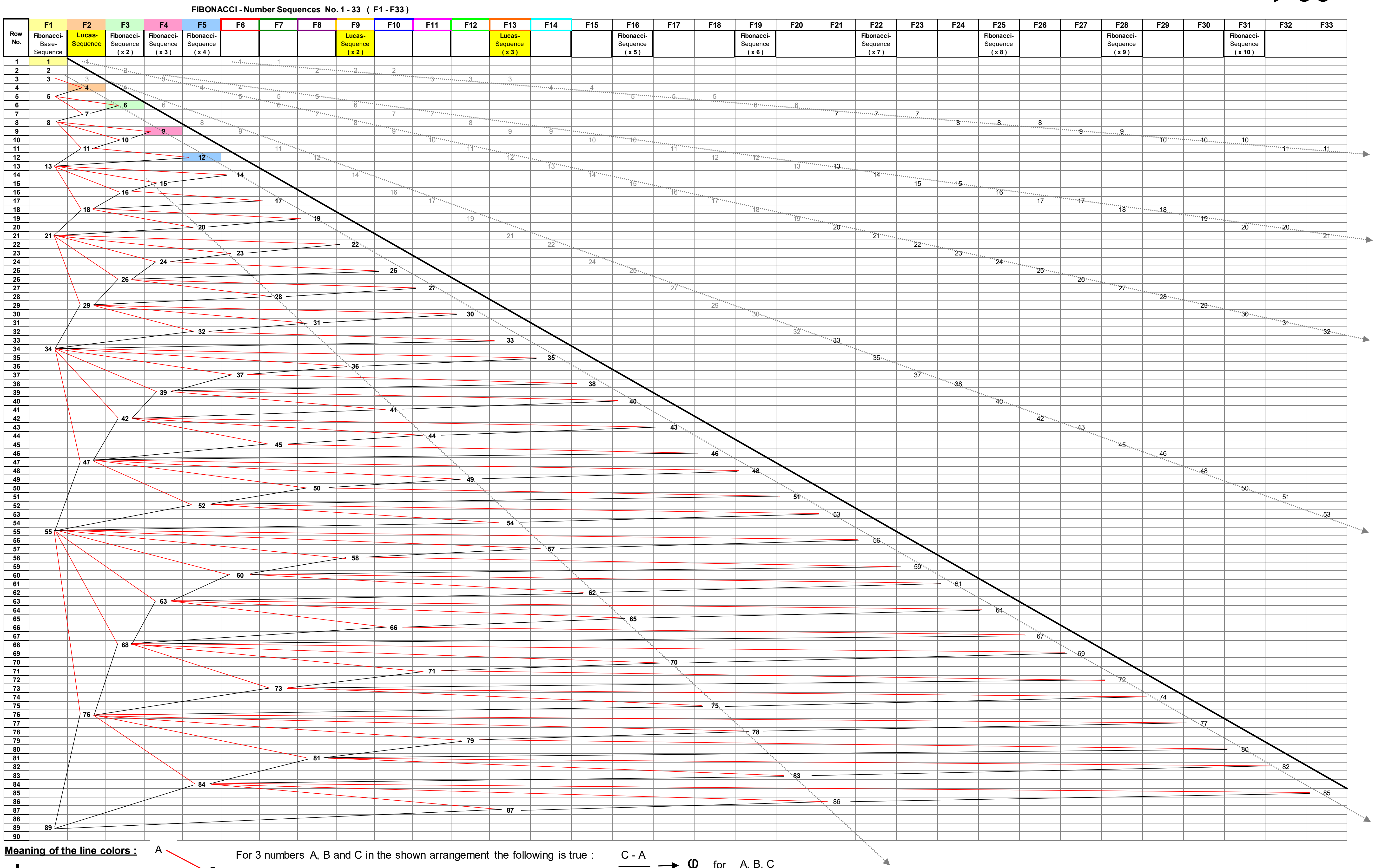
The Ordered Distribution of Natural Numbers on the Square Root Spiral - by Harry K. Hahn
http://front.math.ucdavis.edu/0712.2184 **PDF :** http://arxiv.org/pdf/0712.2184

The Distribution of Prime Numbers on the Square Root Spiral - by Harry K. Hahn
http://front.math.ucdavis.edu/0801.1441 **PDF :** http://arxiv.org/pdf/0801.1441

Appendix A.) :

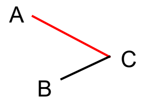
Infinite Fibonacci – Number – Sequence - Table : Sequences No. 1 to 33 shown (F1 – F33) :

→ ∞



Meaning of the line colors :

↓ ∞



For 3 numbers A, B and C in the shown arrangement the following is true :

The ratio of the difference (C-A) indicated by a "red line" to the difference (B-C) indiated by a "black line" is approaching the golden ratio φ for the further progressing number sequences (which contain these numbers) towards infinity (->downwards).

$$\frac{C - A}{B - C} \longrightarrow \varphi \quad \text{for} \quad A, B, C \longrightarrow \infty$$

Appendix B.) : Fibonacci – Sequence – Tables F1 ; F2 ; F6 ; F8

Note: The numbers of the Fibonacci F1 – Number Sequence seem to contain all prime numbers as prime factors ! and all prime factors appear periodic in defined “number-distances“ in the sequence (see left side of table)

Table 2 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F1** - Number Sequence :

some prime factors shown in table form													in prime factors factorized Fibonacci-Numbers		sum of digits	Fibonacci-Sequence F1			
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products	new products		F	F'	F"	Nr.
															1	1			1
															1	1			2
															2	2	1		3
															3	3	1		4
															5	5	2	1	5
												2^3		2x2x2	8	8	3	1	6
															4	13	5	2	7
									7		3			3x7	3	21	8	3	8
						17						2		2x17	7	34	13	5	9
								11		5				5x11	10	55	21	8	10
															17	89	34	13	11
											3^2	2^4	2x2x2	2x3x3	9	144	55	21	12
															8	233	89	34	13
			29				13							13x29	17	377	144	55	14
										5		2		2x5x61	7	610	233	89	15
									7		3		3x7x	47	24	987	377	144	16
															22	1597	610	233	17
					19	17						2^3	2x17x	2x2x19	19	2584	987	377	18
	37													37x113	14	4181	1597	610	19
41								11		5	3		5x11x	3x41	24	6765	2584	987	20
							13					2		2x13x421	20	10946	4181	1597	21
														89x199	17	17711	6765	2584	22
															28	28657	10946	4181	23
				23					7		3^2	2^5	2x2x2x2x3x3x	2x7x23	27	46368	17711	6765	24
										5^2				5x5x3001	19	75025	28657	10946	25
														233x521	19	121393	46368	17711	26
						17						2		2x17x53x109	29	196418	75025	28657	27
			29				13				3		13x29x	3x281	21	317811	121393	46368	28
															23	514229	196418	75025	29
		31						11		5		2^3	2x5x61x	2x2x11x31	17	832040	317811	121393	30
														557x2417	31	1346269	514229	196418	31
									7		3		3x7x47x	2207	30	2178309	832040	317811	32
												2		2x89x19801	34	3524578	1346269	514229	33
														1597x3571	37	5702887	2178309	832040	34
							13			5				5x13x141961	35	9227465	3524578	1346269	35
					19	17					3^3	2^4	2x2x2x17x19x	2x3x3x3x107	27	14930352	5702887	2178309	36
														73x149x2221	35	24157817	9227465	3524578	37
	37												37x113x	9349	44	39088169	14930352	5702887	38
												2		2x233x135721	43	63245986	24157817	9227465	39
41								11	7	5	3		3x5x11x41x	7x2161	24	102334155	39088169	14930352	40
														2789x59369	31	165580141	63245986	24157817	41
			29				13					2^3	2x13x421x	2x2x29x211	46	267914296	102334155	39088169	42
															41	433494437	165580141	63245986	43
											3		89x199x	3x43x307	33	701408733	267914296	102334155	44
						17				5		2		2x5x17x61x109441	29	1134903170	433494437	165580141	45
														139x461x28657	35	1836311903	701408733	267914296	46
															37	2971215073	1134903170	433494437	47
				23					7		3^2	2^6	2x2x2x2x2x3x3x7x23x	2x47x1103	54	4807526976	1836311903	701408733	48

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange

Table 3 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F2 (Lucas)** - Number Sequence :

some prime factors shown in table form													in prime factors factorized Fibonacci-Numbers		sum of digits	**Fibonacci-Sequence F2 (Lucas-Sequence)**			
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products	new products		**L**	L'	L"	**No.**
															1	1			**1**
															3	3			**2**
												2^2		2x2	4	4	1		**3**
															7	7	3		**4**
															4	11	4	1	**5**
											3^2	2		2x3x3	9	18	7	3	**6**
															11	29	11	4	**7**
															11	47	18	7	**8**
					19							2^2	2x2x	19	13	76	29	11	**9**
41											3			3x41	6	123	47	18	**10**
															19	199	76	29	**11**
				23					7			2		2x7x23	7	322	123	47	**12**
															8	521	199	76	**13**
											3			3x281	15	843	322	123	**14**
		31						11				2^2		2x2x11x31	14	1364	521	199	**15**
															11	2207	843	322	**16**
															16	3571	1364	521	**17**
											3^3	2	2x3x3x	3x107	27	5778	2207	843	**18**
															25	9349	3571	1364	**19**
									7					7x2161	16	15127	5778	2207	**20**
			29									2^2		2x2x29x211	23	24476	9349	3571	**21**
											3			3x43x307	21	39603	15127	5778	**22**
														139x461	26	64079	24476	9349	**23**
												2		2x47x1103	20	103682	39603	15127	**24**
	Prime factor is missing					**Prime factor is missing**	**Prime factor is missing**	11		**Prime factor is missing**				11x101x151	28	167761	64079	24476	**25**
											3			3x90481	21	271443	103682	39603	**26**
					19							2^2	2x2x19x	5779	22	439204	167761	64079	**27**
									7					7x7x14503	25	710647	271443	103682	**28**
														59x19489	29	1149851	439204	167761	**29**
41											3^2	2	3x41x	2x3x2521	36	1860498	710647	271443	**30**
															20	3010349	1149851	439204	**31**
														1087x4481	38	4870847	1860498	710647	**32**
												2^2		2x2x199x9901	40	7881196	3010349	1149851	**33**
											3			3x67x63443	24	12752043	4870847	1860498	**34**
			29					11						11x29x71x911	28	20633239	7881196	3010349	**35**
				23					7			2	2x7x23x	103681	34	33385282	12752043	4870847	**36**
															26	54018521	20633239	7881196	**37**
											3			3x29134601	33	87403803	33385282	12752043	**38**
												2^2		2x2x79x521x859	23	141422324	54018521	20633239	**39**
														47x1601x3041	38	228826127	87403803	33385282	**40**
															34	370248451	141422324	54018521	**41**
											3^2	2	3x281x	2x3x83x1427	54	599074578	228826127	87403803	**42**
														6709x144481	43	969323029	370248451	141422324	**43**
									7					7x263x881x967	52	1568397607	599074578	228826127	**44**
		31			19			11				2^2	2x2x11x31x	19x181x541	41	2537720636	969323029	370248451	**45**
											3			3x4969x275449	30	4106118243	1568397607	599074578	**46**
															62	6643838879	2537720636	969323029	**47**
												2		2x769x2207x3167	47	10749957122	4106118243	1568397607	**48**
			29											29x599786069	46	17393796001	6643838879	2537720636	**49**
41											3			3x41x401x570601	39	28143753123	10749957122	4106118243	**50**

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange

Table 4 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F6** - Number Sequence :

Periodicity of the prime factors **2 - 41** shown in table form													in prime factors factorized Fibonacci-(F6)-Numbers	sum of digits	**Fibonacci-F6 Sequence**			
41	37	31	29	23	19	17	13	11	7	5	3	2			**F6**	F6'	F6"	**Nr.**
???			prime factor is missing			prime factor is missing	prime factor is missing	prime factor is missing							1			**1**
												2^2	2x2		4			**2**
															5	1		**3**
											3^2		3x3		9	4		**4**
									7			2	2x7		14	5	1	**5**
															23	9	4	**6**
															37	14	5	**7**
										5	3	2^2	2x2x3x5		60	23	9	**8**
															97	37	14	**9**
															157	60	23	**10**
												2	2x127		254	97	37	**11**
											3		3x137		411	157	60	**12**
					19				7	5			5x7x19		665	254	97	**13**
												2^2	2x2x269		1076	411	157	**14**
															1741	665	254	**15**
											3^2		3x3x313		2817	1076	411	**16**
												2	2x43x53		4558	1741	665	**17**
										5^3			5x5x5x59		7375	2817	1076	**18**
															11933	4558	1741	**19**
											3	2^2	2x2x3x1609		19308	7375	2817	**20**
									7				7x4463		31241	11933	4558	**21**
															50549	19308	7375	**22**
										5		2	2x5x8179		81790	31241	11933	**23**
		31									3		3x31x1423		132339	50549	19308	**24**
															214129	81790	31241	**25**
	37											2^2	2x2x37x2341		346468	132339	50549	**26**
															560597	214129	81790	**27**
										5	3^2		3x3x5x6719		907065	346468	132339	**28**
									7			2	2x7x79x1327		1467662	560597	214129	**29**
				23									23x223x463		2374727	907065	346468	**30**
					19								19x202231		3842389	1467662	560597	**31**
											3	2^2	2x2x3x379x1367		6217116	2374727	907065	**32**
										5			5x227x8863		10059505	3842389	1467662	**33**
															16276621	6217116	2374727	**34**
												2	2x641x20543		26336126	10059505	3842389	**35**
											3		3x1637x8677		42612747	16276621	6217116	**36**
									7				7x181x54419		68948873	26336126	10059505	**37**
										5		2^2	2x2x5x5578081		111561620	42612747	16276621	**38**
															180510493	68948873	26336126	**39**
											3^2		3x3x32452457		292072113	111561620	42612747	**40**
												2	2x1109x213067		472582606	180510493	68948873	**41**
													67x2083x5479		764654719	292072113	111561620	**42**
										5^2			5x5x49489493		1237237325	472582606	180510493	**43**
											3	2^2	2x2x3x53x3147629		2001892044	764654719	292072113	**44**
	37								7^2				7x7x37x1786613		3239129369	1237237325	472582606	**45**
													71x3613x20431		5241021413	2001892044	764654719	**46**
												2	2x167x3607x7039		8480150782	3239129369	1237237325	**47**
										5	3		3x5x914744813		13721172195	5241021413	2001892044	**48**
					19								19x83x14078201		22201322977	8480150782	3239129369	**49**
												2^2	2x2x337x2664083		35922495172	13721172195	5241021413	**50**
													129631x448379		58123818149	22201322977	8480150782	**51**
											3^2		3x3x2671x3912239		94046313321	35922495172	13721172195	**52**
									7	5		2	2x5x7x2173859021		152170131470	58123818149	22201322977	**53**
		31		23									23x31x345324607		246216444791	94046313321	35922495172	**54**
															398386576261	152170131470	58123818149	**55**

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange

Table 5 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F8** - Number Sequence :

Periodicity of the prime factors **2 - 41** shown in table form													in prime factors factorized Fibonacci-(F8)-Numbers	sum of digits	**Fibonacci-F8 Sequence**			
41	37	31	29	23	19	17	13	11	7	5	3	2			**F8**	F8'	F8"	**Nr.**
															2			**1**
															5			**2**
															7	2		**3**
												2^2	2x2x3		12	5		**4**
															19	7	2	**5**
															31	12	5	**6**
										5^2		2	2x5x5		50	19	7	**7**
											3^4		3x3x3x3		81	31	12	**8**
															131	50	19	**9**
												2^2	2x2x53		212	81	31	**10**
									7^3				7x7x7		343	131	50	**11**
	37									5	3		3x5x37		555	212	81	**12**
												2	2x449		898	343	131	**13**
															1453	555	212	**14**
															2351	898	343	**15**
											3	2^2	2x2x3x317		3804	1453	555	**16**
										5			5x1231		6155	2351	898	**17**
				23									23x433		9959	3804	1453	**18**
									7			2	2x7x1151		16114	6155	2351	**19**
											3^2		3x3x2897		26073	9959	3804	**20**
															42187	16114	6155	**21**
										5		2^2	2x2x5x3413		68260	26073	9959	**22**
???			???		19	prime factor is missing	prime factor is missing	prime factor is missing					19x5813		110447	42187	16114	**23**
											3		3x71x839		178707	68260	26073	**24**
												2	2x144577		289154	110447	42187	**25**
													67x6983		467861	178707	68260	**26**
									7	5			5x7x43x503		757015	289154	110447	**27**
											3	2^2	2x2x3x103x991		1224876	467861	178707	**28**
															1981891	757015	289154	**29**
															3206767	1224876	467861	**30**
	37											2	2x37x70117		5188658	1981891	757015	**31**
										5^2	3^2		3x3x5x5x37313		8395425	3206767	1224876	**32**
															13584083	5188658	1981891	**33**
												2^2	2x2x397x13841		21979508	8395425	3206767	**34**
									7				7x83x61211		35563591	13584083	5188658	**35**
		31									3		3x31x401x1543		57543099	21979508	8395425	**36**
										5		2	2x5x53x175673		93106690	35563591	13584083	**37**
													6257x24077		150649789	57543099	21979508	**38**
													919x265241		243756479	93106690	35563591	**39**
											3	2^2	2x2x3x59x97x5743		394406268	150649789	57543099	**40**
					19								19x33587513		638162747	243756479	93106690	**41**
				23						5			5x23x229x39209		1032569015	394406268	150649789	**42**
									7			2	2x7x2677x44579		1670731762	638162747	243756479	**43**
											3^3		3x3x3x599x167149		2703300777	1032569015	394406268	**44**
													2693x1624223		4374032539	1670731762	638162747	**45**

Note : **all prime numbers are marked in yellow** **and all numbers not divisible by 2, 3 or 5 are marked in orange**