

# Natural Numbers and their Square Roots expressed by constant Phi and 1

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## Abstract :

All natural numbers ( 1, 2, 3,...) can be calculated only by using constant Phi ( $\varphi$ ) and 1.

I have found a way to express all natural numbers and their square roots with simple algebraic terms, which are only based on Phi ( $\varphi$ ) and 1. Further I have found a rule to calculate all natural numbers >10 and their square roots with the help of a general algebraic term. The constant Pi ( $\pi$ ) can also be expressed only by using the constant  $\varphi$  and 1 !

## Introduction :

### The asymptotic ratio of successive Fibonacci numbers leads to the golden ratio constant $\varphi$ ( or $\Phi$ )

Fibonacci Sequences describe morphological patterns in a wide range of living organisms. This is one of the most remarkable organizing principles mathematically describing natural phenomena.

The constant  $\varphi$  is the positive solution of the following quadratic equation :

$$x + 1 = x^2$$

$$\rightarrow \varphi = \frac{1 + \sqrt{5}}{2} = 1.618034\dots$$

### The Fibonacci Numbers

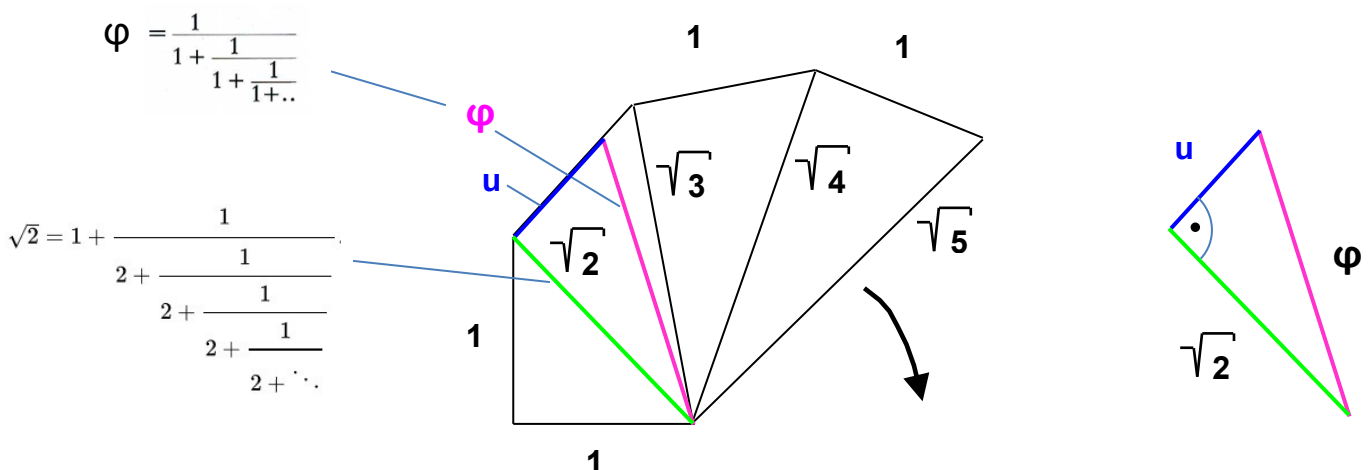
defined by  $\varphi$  :

1/1 = 1  
 2/1 = 2  
 3/2 = 1.5  
 5/3 = 1.667  
 8/5 = 1.6  
 13/8 = 1.625  
 21/13 = 1.615  
 34/21 = 1.619  
 55/34 = 1.618



Because the value of constant  $\varphi$  is close to the square root of 2 and the square root of 3 , I have drawn  $\varphi$  into the start section of the **Square Root Spiral** in order to find a way to calculate the short cathetus  $u$  of the right triangle  $\varphi$  , square root of 2 and  $u$  , and to see which relation the cathetus  $u$  has to the other triangles of the Square Root Spiral :

### The start of the Square Root Spiral is shown with the constant $\varphi$ drawn in :



The periodic continued fractions of  $\varphi$  and square root of 2 show a very simple structure. But what is with cathetus  $u$  ?

Now I calculated the numerical value of chatetus  $\mathbf{u}$  with the help of the **Pythagorean Theorem** :

From the right triangle  $\varphi$  , square root of  $\mathbf{2}$  &  $\mathbf{u}$  follows :

$$\varphi^2 = (\sqrt{2})^2 + u^2 \quad ; \quad \text{application of the Pythagorean Theorem}$$

$$\rightarrow u = \sqrt{\varphi^2 - 2} = 0,786151377..... \quad ; \quad \text{we can calculate this value of } u \text{ with the calculator}$$

But because this numerical value doesn't say much, I did some research in the internet with Google, and I actually found an algebraic term which obviously has the same numerical value !

This is the following term :

$$\frac{\sqrt{2\sqrt{5} - 2}}{2} = 0,786151377... = u$$

This value is shown in **equation 4.10. on page 11** of the following study :

Title of this study : „**PHASE SPACES IN SPECIAL RELATIVITY : TOWARDS ELIMINATING GRAVITATIONAL SINGULARITIES**“

by Peter Danenhower - **weblink** : <https://arxiv.org/pdf/0706.2043.pdf>

**Also read this study !**: [The Black Hole in M87 \(EHT2017\) may provide evidence for a Poincare Dodecahedral Space Universe](#)

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**With the help of the found algebraic term I carried out the following algebraic calculations :**

$$\sqrt{\varphi^2 - 2} = \frac{\sqrt{2\sqrt{5} - 2}}{2} \quad ; \quad \text{I equated the two algebraic terms which obviously represent the same constant !}$$

$$\rightarrow 4\varphi^2 - 8 = 2\sqrt{5} - 2 \quad ; \quad \text{I squared both sides and transformed}$$

$$\varphi^2 = \frac{\sqrt{5} + 3}{2} \quad ; \quad (1) \quad \text{I solved for } \varphi^2$$

$$\sqrt{5} = 2\varphi^2 - 3 \quad ; \quad (2) \quad \text{I solved for } \sqrt{5}$$

Now I went back to the Square Root Spiral and used the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem}$$

$$6 = (2\varphi^2 - 3)^2 + 1 \quad ; \quad \text{I replaced } \sqrt{5} \text{ by equation (2) and transformed}$$

$$\rightarrow 3 = \frac{\varphi^4 + 1}{\varphi^2} \quad (3) \quad \rightarrow \quad \sqrt{3} = \sqrt{\frac{\varphi^4 + 1}{\varphi^2}} \quad (4) \quad ; \quad \text{square root 3 expressed by } \varphi \text{ and 1 !}$$

Now I used the following right triangle :

$$(\sqrt{3})^2 = (\sqrt{2})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem and inserting equation ( 3 )}$$

$$\rightarrow 2 = \frac{\varphi^4 + 1}{\varphi^2} - 1 \quad \rightarrow \quad 2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} \quad (5) \quad \text{and} \quad \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \quad (6)$$

Then I inserted equation ( 3 ) in equation ( 2 ) :

$$\rightarrow \sqrt{5} = 2\varphi^2 - \frac{\varphi^4 + 1}{\varphi^2} \quad \rightarrow \quad \sqrt{5} = \frac{\varphi^4 - 1}{\varphi^2} \quad ; \quad (7.0) \quad \rightarrow \quad 5 = \left( \frac{\varphi^4 - 1}{\varphi^2} \right)^2 \quad (7.1)$$

And I used the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem and inserting equation ( 7.1 )}$$

$$\rightarrow 6 = \left( \frac{\varphi^4 - 1}{\varphi^2} \right)^2 + 1 \quad \rightarrow \quad 6 = \frac{\varphi^8 - \varphi^4 + 1}{\varphi^4} \quad (8) \quad \text{and} \quad \sqrt{6} = \sqrt{\frac{\varphi^8 - \varphi^4 + 1}{\varphi^4}} \quad (9)$$

I continued and used the following right triangles of the **Square Root Spiral (SRS)** to calculate the next square roots :

$$(\sqrt{7})^2 = (\sqrt{6})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem and inserting equation ( 8 )}$$

$$\rightarrow 7 = \frac{\varphi^8 + 1}{\varphi^4} \quad (10) \quad \rightarrow \quad \sqrt{7} = \sqrt{\frac{\varphi^8 + 1}{\varphi^4}} \quad (11)$$

In the same way I calculated the following square roots and natural numbers with the next right triangles of the **SRS** :

$$\rightarrow 8 = \frac{\varphi^8 + \varphi^4 + 1}{\varphi^4} \quad (12) \quad \text{and} \quad \sqrt{8} = \sqrt{\frac{\varphi^8 + \varphi^4 + 1}{\varphi^4}} \quad (13)$$

$$\rightarrow 10 = \frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4} \quad (14) \quad \text{and} \quad \sqrt{10} = \sqrt{\frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4}} \quad (15)$$

$$\rightarrow 11 = \frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4} \quad (16) \quad \text{and} \quad \sqrt{11} = \sqrt{\frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4}} \quad (17)$$

$$\rightarrow 12 = \frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4} \quad (18) \quad \text{and} \quad \sqrt{12} = \sqrt{\frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4}} \quad (19)$$

From the above shown formulas ( equations 12 to 19 ), I realized a general rule for all Natural Numbers > 10 :

$$\rightarrow (10+n) = \frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4} \quad (20) \quad \text{and} \quad \sqrt{(10+n)} = \sqrt{\frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4}} \quad (30)$$

For  $n \rightarrow \infty$

with  $n \in \mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$

**Note:** → The expression (3+n) in the rule can be replaced by products and/or sums, of the equations ( 3 ) to ( 13 ) and number 1, in order to have final expressions only based on  $\varphi$  and 1 !

With these general equations (20) and (30) all natural numbers and their square roots can be expressed by only using constant  $\varphi$  and 1 !

The constant Pi ( $\pi$ ) can also be expressed by only using the constant  $\varphi$  and 1 ! :

I use Viète's formula from the year 1593 : → It is also possible to derive from Viète's formula a related formula for  $\pi$  that involves nested square roots of two, but uses only one multiplication :

$$\pi = \frac{2}{\sqrt{2}} \frac{2}{\sqrt{2+\sqrt{2}}} \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \dots$$

$$\pi = \lim_{k \rightarrow \infty} 2^k \underbrace{\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}}}}}_{k \text{ square roots}}$$

I replace the number 2 in the above shown formulas by the found equation ( 5 ) where number 2 can be expressed by constant  $\varphi$  and 1. Then the constant Pi ( $\pi$ ) can be expressed by only using the constant  $\varphi$  and 1 !

I replaced Number 2 in the above shown formula on the righthand side, with equation ( 5 ) :

$$\pi = \lim_{k \rightarrow \infty} \left[ \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} \right]^k \underbrace{\sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} - \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} + \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} + \dots + \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}}}}_{k \text{ square roots}} \quad (40)$$

It seems that the irrationality of Pi ( $\pi$ ) is fundamentally based on the constant  $\varphi$  and 1, in the same way as the irrationality of all irrational square roots, and all natural numbers seems to be based on constant  $\varphi$  & 1 !

This is an interesting discovery because it allows to describe many basic geometrical objects like the Platonic Solids only with  $\varphi$  & 1 !

Constant  $\varphi$  and Number 1 ( the base unit ) may represent something like fundamental „space structure constants“ !

**Referring to my discovery regarding constant  $\varphi$  (Phi), I want to define the following 12 Conjectures :**

**Here the 12 conjectures :** (  $\rightarrow$  you can call them **Harry K. Hahn's conjectures** )

1.) All Natural Numbers and their square roots can be expressed (calculated) by only using the mathematical constant Phi (golden mean = 1.618..) and number 1. This statement is also valid for all rationals (fractions) and their square roots

2.) All existing irrational numbers seem to be constructions out of Phi and 1.

For example the irrational transcendental constant Pi (3.1415926....) can also be expressed by only using Phi and 1 !

3.) Phi and 1 are the base units of Mathematics ! Numbers and number-systems don't exist ! They are manmade and therefore can be eliminated. In principle Mathematical Science can be carried out by only using Phi and 1, as base units.

4.) All geometrical objects, including the Platonic Solids can be described by only using constant Phi and 1.

Because all natural numbers, their square roots, rationals (fractions) and probably all irrational and all transcendental numbers too, can be expressed by only using Phi and 1.

5.) Point 4.) leads me to the conclusion that in the physical world the geometries of all possible crystal-lattice-structures are fundamentally based on Phi and 1. The more fundamental the lattice the simpler it can be expressed by Phi and 1.

6.) Point 4.) 5.) & 7.) leads me to the conclusion that on the molecular level (and probably on the atomic level too), as well as on the macroscopic (cosmic) level the distribution and structure of matter (=energy) in space, is fundamentally based on constant Phi and 1.  $\rightarrow$  **Phi represents a fundamental physical „Space Structure Constant“**

Together with Point 7.) this indicates that the curvature of spacetime at the molecular level (crystals) and at the atomic level, as well as on the macroscopic level is defined only by the "Space Structure Constant Phi" and the base unit 1.  $\rightarrow$  This idea will help to unify General Relativity with Quantum Mechanics ! If the gravitational singularity in M87 indeed has a dodecahedral structure then gravitation, which is the geometric property of spacetime, can be described in Quantum Mechanics and at the cosmic level by the same constant duo : Phi and base unit 1 !

7.) The structure of the M87 black hole ( $\rightarrow$  **EHT2017**) indicates a dodecahedral structure. The distribution of matter in gravitational singularities therefore seems to be defined essentially by constant Phi and base unit 1 ! The largescale distribution of matter in the universe seems to be predominantly based on an order-5 Poincare-Dodecahedral-Space.  $\rightarrow$  [weblink to my study](#) (or alternatively here : <http://vixra.org/abs/1907.0348>)

**Title : "EHT2017 may provide evidence for a Poincare Dodecahedral Space Universe"**

8.) The natural numbers can be assigned to a defined infinite set of Fibonacci-Number Sequences.

9.) This infinite set of Fibonacci-Number Sequences, and the numbers contained in these sequences, are connected to each other by a complex precisely defined spatial network based on constant Phi. For the progressing Fibonacci-Sequences towards infinity, the connections between the numbers approach constant Phi.

$\rightarrow$  **see my study about the creation of the "Infinite Fibonacci Number Sequence Table"**

10.) Constant Phi (golden mean = 1.618..) must be a fundamental constant of the final equation(s) of the universal mathematical and physical theory. ( $\rightarrow$  It may be the only irrational constant that appears in the(se) equation(s) )

11.) The number-5-oscillation ( $\rightarrow$  the numbers divisible by 5) in the two number sequences  $6n+5$  (Sequence 1) and  $6n+1$  (Sequence 2), with  $n=(0,1,2,3,...)$ , defines the distribution of the prime numbers and non-prime-numbers. The number-5-oscillation defines the starting point and the wave length of defined non-prime-number-oscillations in these Sequences 1+2 (SQ1 & SQ2). (Note : the combination of the two sequences SQ1 & SQ2 is considered here )

$\rightarrow$  weblink to my study : <https://arxiv.org/abs/0801.4049> (or alternatively here : <http://vixra.org/abs/1907.0355>)

For a quick overview please see **pages 15 to 18** in this study : [weblink to the study](#) : **"EHT2017 may provide evidence..."**

12.) The importance of the number-5-oscillation for the distribution of primes and non-primes is a further indication for the conjecture that the largescale structure of the universe seems to be predominantly (mainly) based on an order-5 Poincare-Dodecahedral-Space structure.  $\rightarrow$  The space structure of the universe seems to be based essentially on the **5.Platonic Solid : the Dodecahedron** ( $\rightarrow$  consisting of 12 regular pentagonal faces, three faces meeting at each vertex)

**The time will show if my Conjectures are correct !**

## References :

**Phase spaces in Special Relativity : Towards eliminating gravitational singularities** - by Peter Danenhowe

see weblink : <https://arxiv.org/pdf/0706.2043.pdf>

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## further interesting References to the subject :

**The Black Hole in M87 (EHT2017) may provide evidence for a Poincare Dodecahedral Space Universe** - by Harry K. Hahn

<https://archive.org/details/TheBlackHoleInM87EHT2017MayProvideEvidenceForAPoincareDodecahedralSpaceUniverse/page/n1>

**The Ordered Distribution of Natural Numbers on the Square Root Spiral** - by Harry K. Hahn

<http://front.math.ucdavis.edu/0712.2184> PDF : <http://arxiv.org/pdf/0712.2184>

**The Distribution of Prime Numbers on the Square Root Spiral** – by Harry K. Hahn

<http://front.math.ucdavis.edu/0801.1441> PDF : <http://arxiv.org/pdf/0801.1441>

**The golden ratio Phi ( $\varphi$ ) in Platonic Solids:** <http://www.sacred-geometry.es/?q=en/content/phi-sacred-solids>

**Number Theory as the Ultimate Physical Theory** - by I. V. Volovich / Steklov Mathematical Institute

Study : <http://cdsweb.cern.ch/record/179558/files/198708102.pdf>

**Letters of Albert Einstein, including his letter to natural constants from 13th October 1945** ( in german language )

<http://docplayer.org/69639849-Ilse-rosenthal-schneider-begegnungen-mit-einstein-von-laue-und-planck.html>

description of the book contents in english : <http://blog.alexander-unzicker.com/?p=27>